A MARKOVIAN RELIABILITY ANALYSIS OF ELECTRONIC CIRCUITS

by

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INTRODUCTION

In recent years reliability has become everybody's business. The development of reliability originated in the early part of the year 1958, when the United States suffered a temporary setback in her attempt to place an unmanned satellite into orbit, after Russia launched her first space satellite, Sputnik I, in November, 1957. The series of failures and subsequent dismay and loss in prestige were very serious. Several billions of dollars and years of development and research were reduced to nothing as a result of the unreliability of the system.

Although it is possible mathematically to formulate a model to partially explain the unreliability of a system, there are such factors as human beings, communications, understanding and requirements which cannot be readily quantified and included in the expression for reliability. These intangible factors should not, however, be ignored altogether. The model we shall create will base its usefulness on how well the assumptions coincide with reality and on the facility with which the life test data can be used to evaluate the measures of performance.

This report deals with the evaluation of circuit reliability from the knowledge of experimental data on component parts. The model developed here will take into account the following modes of failure:

(1) Failure of the circuit by the simultaneous drift of the component parameters from their nominal values.

(2) Failure of the circuit by the catastrophic failure of a constituent part.

The need for the development of the model arose when the models hitherto used failed to provide for two significant characteristics of the circuit behavior. These are:

(1) A circuit considered operative at a time point 't_2' may have failed earlier at 't_1' and then drifted back into an operative region by the time 't_2.' The immediate past history of each component part cannot be ignored.

(2) A finite state, continuous-time Markov process can explain more accurately the circuit behavior than a finite state, discrete-time Markov process, as circuit deterioration occurs over a continuous scale of time.

The analysis presented here is both exhaustive and comprehensive. The report has the following chronological break down:

(1) Definitions, assumptions, and characteristics of the model.

(2) The structure of the process that will be used to represent the component behavior.

(3) The derivation of the circuit processes from the component processes.

(4) The derivation of the reliability equation from the circuit process synthesised from the component processes.

(5) Estimation, testing, and correlation of Markov chains.

(6) Application of the Markovian model to a simple two resistor circuit.

(7) Discussion.

DEFINITIONS, ASSUMPTIONS, AND CHARACTERISTICS OF THE MODEL

DEFINITIONS

COMPONENT STATE: A component parameter may assume any value in its range of values. This operable range may be further partitioned into several disjoint intervals. The state of the component at a time point 't' is then the set to which the value of the component parameter is assigned.

CIRCUIT STATE: Several components constitute a circuit. Once each of the components have been partitioned into disjoint intervals, it is possible to synthesise the circuit states of the components. A circuit state at time 't' is thus a K-tuple of the component parameter states at time 't.'

INFINITESIMAL GENERATOR MATRIX: Λ is a matrix called the infinitesimal generator matrix when its elements λ_{ij} are so defined that $\lambda_{ij}\ \Delta t + 0\ (\Delta t)$, $i \neq j$, is the probability that a system in state 'i' at time zero enters state 'j' by time Δt with $\lambda_{ii} = -\sum_{i \neq j} \lambda_{ij}$.

FINITE STATE, DISCRETE TIME, MARKOV PROCESS OF ORDER 'Z': If we denote the state of the system at time 't' by a random variable, x(t), then the distribution of x(t) is a finite state, discrete time, Markov process of order 'Z' if, and only if,

(1) x(t) can assume finite values.

(2) $P_r \left\{ x(t) = b_t \,/\, X(t-1) = b_{t-1},\ X(t-2) = b_{t-2},\ \ X(0) = b_o \right\}$

$$= P_r \left\{ X(t) = b_t \,/\, X(t-1) = b_{t-1}, X(t-2) = b_{t-2}, \ldots, X(t-Z) = b_{t-Z} \right\}$$

where b_t = state of the component part at time 't.'

For $Z = 1$,

$$P_r \left\{ X(t) = b_t / X(t-1) = b_{t-1}, X(t-2) = b_{t-2}, \ldots, X(0) = b_o \right\}$$

$$= P_r \left\{ X(t) = b_t \,/\, X(t-1) = b_{t-1} \right\}$$

STATIONARY TRANSITION PROBABILITIES: The transition probabilities $p_{ij}(t)$ are stationary if, and only if, $p_{ij}(t) = p_{ij}$, a constant, for all values of 't.' This implies that given that the system is in state 'i' at time 't', the probability that the system will occupy state 'j' at some future time is independent of 't.'

COMPONENT PART: A component part is an element of the circuit. Example: Resistors R1, R2, R3, R4, etc.

ASSUMPTIONS AND CHARACTERISTICS

COMPONENT PARAMETER VARIATION: It will be assumed that the transition of one parameter of the circuit is independent of the corresponding transitions of the other parameters, given that these transitions are random with respect to time.

COMPONENT PARAMETER BEHAVIOR: It is assumed that the state of the component parameter at some future time, given the state of the component parameter at the present time, is independent of the present age of the component, the previous states the component parameter has occupied the time spent in each of the previous states, and the time spent in the present state, but is entirely dependent only on the time difference. The distribution of the component parameter variation can be represented by a discrete state, continuous time, Markov process with stationary transition probabilities.

FAILURE SPECIFICATION FOR CIRCUIT: It will be assumed that there exists a range for the output parameter of the circuit, for which the circuit is considered reliable. Such a specification enables one to partition the total number of circuit states into two disjoint sets, namely, the operative and failure sets.

COMPONENT LIFE TEST DATA: Assume that the range of values of the parameters is divided arbitrarily into a finite number of states. Of the N statistically identical parts put on test, let $N_{ij}(t)$ be the number of parameters observed in state 'i' at the $(t-1)^{st}$ period and observed in state 'j' at the t^{th} period. A large and extensive life test data at various time points must be available in $N_{ij}(t)$ for every component parameter.

THE STRUCTURE OF THE PROCESS OF THE COMPONENT PARAMETER VARIATION

Let $JSR1_{(n)}$ represent the state of the process of the variation of the component parameter R_1, after n transitions. Then $JSR1_{(0)}$, $JSR1_{(1)}, \ldots, JSR1_{(n)}$ will represent the chronological succession of states of the component parameter R1 after 0, 1, ..., n transitions respectively.

Let the range of R1 be partitioned into 'm' disjoint intervals or states. Then $JSR1_{(n)}$ is a random variable which can take on any of the 'm' values.

At time t=0, the probability of initially starting the process from each of the 'm' possible states is given by

$$a_o^{(i)} \triangleq P_r \left\{ JSR1_{(0)} = i \right\},$$

$$i = 1, 2, \ldots m$$

where $0 \leq a_o^{(i)} \leq 1$, and subject to the condition,

$$\sum_i a_o^{(i)} = 1.$$

Let p_{ij} represent the stationary probability of transition, from state 'i' after (n-1) transitions to state 'j' after n transitions.

$$p_{ij} \triangleq P_r \left\{ JSR1_{(n)} = J/JSR1_{(n-1)}' = i \right\},$$

$$i,j = 1, 2, \ldots, m.$$

where $p_{ij} \geq 0$, and subject to the condition,

$$\sum_{i,j=1}^{m} P_{ij} = 1$$

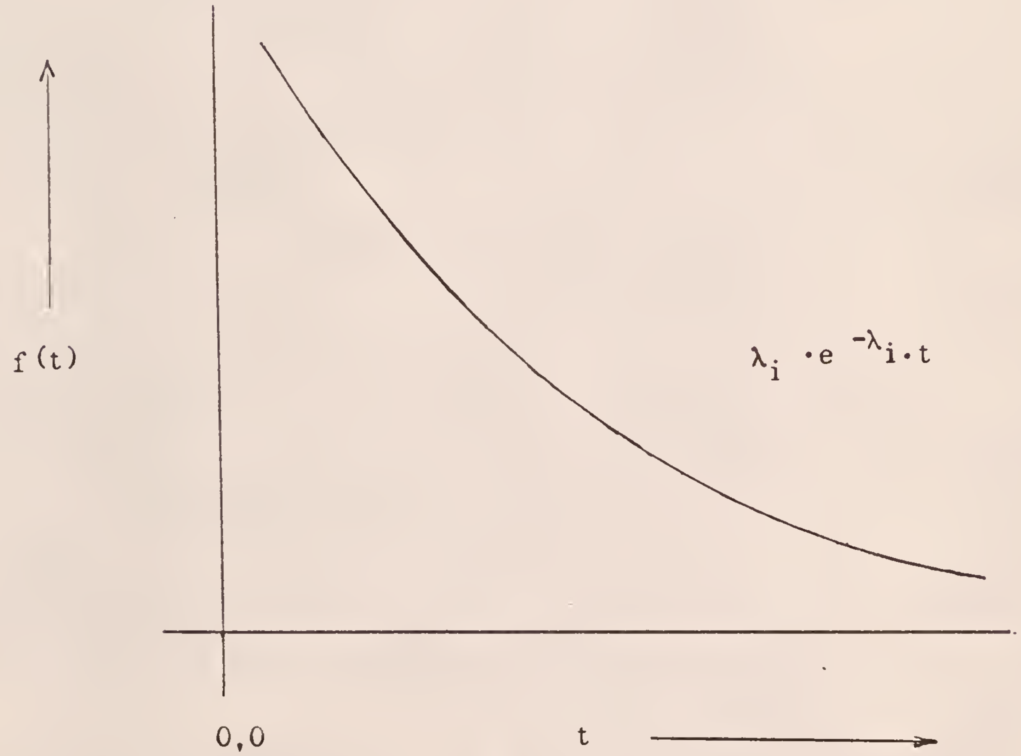
THE DISTRIBUTION OF THE TIME SPENT IN A PARTICULAR STATE

The time spent in a state is a random variable T_n given by,

$$H_i(t) \triangleq P_r \left\{ T_n \leq t \,/\, JSR1_{(n-1)} = i \right\},$$

where $H_i(t)$ is the probability of leaving state 'i' at or before time 't'.

For any given state, it can be proved (4) that the probability that the process is in that state at any time is distributed exponentially.



If $f(t)$ is the probability of occupying state 'i' at time 't,' with a mean time of $(\lambda_i)^{-1}$, then

$$f(t) = \lambda_{i \cdot e}{}^{-\lambda_i \cdot t}, \quad t > 0.$$

$$= 0 \qquad \text{otherwise,}$$

$$H_i(t) = \int_0^t f(t) \cdot dt = \int_0^t \lambda_{i \cdot e}{}^{-\lambda_i \cdot t} = 1 - e^{-\lambda_i \cdot t}$$

If $Q_{ij}(t)$ is the probability of occurence of the joint events, leaving state 'j' at or before time 't' and entering state 'j' after the first transition, then,

$$Q_{ij}(t) = P_r \left\{ T_n \leq t,\ JSR1_{(n)} = j / JSR1_{(n-1)} = i \right\}$$

$$= P_{ij} \ (1 - e^{-\lambda_i \cdot t})$$

The above equation completely identifies the Markov process for the component parameter. The given equation had been arrived at by making two important assumptions. These are :

(1) The distributions of the states and the time spent in each state are independent.

(2) The density function of the time spent in any state in exponentially distributed.

THE DERIVATION OF THE CIRCUIT PROCESS FROM THE COMPONENT PROCESSES

Let $(JSCT)_n$ be the state occupied by the circuit after 'n' transitions.

If the circuit consists of 'k' component parameters, then the state of the circuit is the 'k' tuple of the states of the component parameters.

Corresponding to (JSCT)n, let the component parameters R1, R2,.., Rk, occupy states $(JSR1)_{n_1}$, $(JSR2)_{n_2}$,...,$(JSRk)_{n_k}$ after n_1, n_2,..., and n_k transitions respectively.

Then, $(JSCT)_n = f\left[(JSR1)_{n_1}, (JSR2)_{n_2}, \ldots, (JSRk)_{n_k} \right]$

For simplicity consider a system composed of three parameters R1, R2, and R3.

At time t = 0;

$(JSCT)_0 = i$

$(JSR1)_0 = u$

$(JSR2)_0 = v$

$(JSR3)_0 = w$

Let the circuit state change from state 'i' to state 'j' on the next transition, after a time period t = t. It will be assumed that this change in circuit state is brought about by a change in state of the component parameter R1 only.

At time t = t;

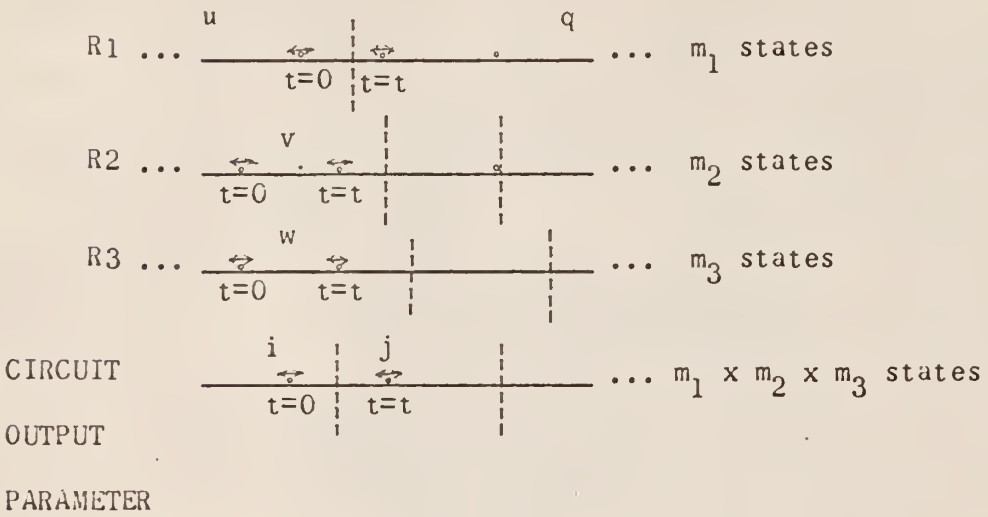
$(JSCT)_1 = j$

$(JSR1)_1 = q$

$(JSR2)_0 = v$

$(JSR3)_0 = w$

The chart shown below schematically illustrates the component transition relative to the circuit transition.



Each of the component parameters R1, R2, and R3 are arbitrarily partitioned into m_1, m_2, and m_3 states respectively. The actual values of each of the parameters before transition (t=0) and after transition (t=t) are represented in the chart.

The notation $\leftrightarrow$ indicates that the transitions can occur in the either direction.

In order that $(JSR1)_0 = u$ changes to $(JSR1)_1 = q$, it is obvious that,

1. Time spent by R1 in state 'u' $(T_{1(R1)}) \leq$ time spent by R2 in state 'v' $(T_{1(R2)})$

2. Time spent by R1 in state 'u' $(T_{1(R1)}) \leq$ time spent by R3 in state 'w' $(T_{1(R3)})$.

3. Time spent by R1 in state 'u' $(T_{1(R1)}) \leq$ time spent by the circuit in 'i' $(T_{1(c)})$.

$$Q_{ij}(t) = \Pr\left\{ T_{1(c)} \leq t,\ (JSCT)_1 = j \,/\, (JSCT)_0 = i \right\}$$

$$= \Pr\left\{ T_{1(R1)} \leq t,\ T_{1(R1)} \leq T_{1(R2)},\ \leq T_{1(R3)},\ (JSR1)_1 = q/ \right.$$

$$\left. (JSR1)_0 = u,\ (JSR2)_0 = v,\ (JSR3)_0 = w \right\}$$

Consider the process, after a small time interval $\tau << t$. For this interval, let $dQ_{ij}(\tau)$ be the infinitesimal probability that the circuit changes from state 'i' to state'j.' In order to obtain the probability that the circuit changes from state 'i' to state 'j' in a time interval $T_{1(c)} \leq t$, it is necessary to integrate all such infinitesimal probabilities over a time interval 't.'

$$dQ_{ij}(\tau) = \left[1 - H_v(\tau) \right] \cdot \left[1 - H_w(\tau) \right] \cdot dQ_{uq}(\tau)$$

Hence,

$$Q_{ij}(\tau) = \int_0^t dQ_{ij}(\tau)$$

$$= \int_0^t \left[1 - H_v(\tau) \right] \cdot \left[1 - H_w(\tau) \right] \cdot dQ_{uq}(\tau)$$

Now,

$$H_v(\tau) = 1 - e^{-\lambda v \cdot \tau}$$

$$H_w(\tau) = 1 - e^{-\lambda w \cdot \tau}$$

$$Q_{uq}(\tau) = P_{uq} \cdot (1 - e^{-\lambda u \cdot \tau})$$

$$dQ_{uq}(\tau) = P_{uq} \cdot \lambda u \cdot e^{-\lambda u \cdot \tau}\, d\tau$$

$$Q_{ij}(t) = \int_0^t e^{-\lambda_v \cdot \tau}\, e^{-\lambda_w \cdot \tau}\, P_{uq} \cdot \lambda u \cdot e^{-\lambda_u \cdot \tau}\, d\tau$$

$$= \int_0^t e^{-(\lambda_u + \lambda_v + \lambda_w) \cdot \tau} \cdot P_{uq} \cdot \lambda_u \cdot d\tau$$

$$= \frac{\lambda_u}{\lambda_u + \lambda_v + \lambda_w} \cdot P_{uq} \cdot \left[-e^{-(\lambda_u + \lambda_v + \lambda_w) \cdot \tau} \right]_0^t$$

$$= \frac{\lambda u}{\lambda_u+\lambda_v+\lambda_w} \cdot P_{uq} \cdot \left[1 - e^{-(\lambda_u+\lambda_v+\lambda_w)\cdot t} \right]$$

Let,

$$\lambda_i = \lambda_u+\lambda_v+\lambda_w,$$

and $$P_{ij} = \frac{\lambda u}{\lambda_u+\lambda_v+\lambda_w} \cdot P_{uq}$$

$$= \frac{\lambda_u}{\lambda_i} \cdot P_{uq}$$

$$Q_{ij}(t) = P_{ij} \left[1 - e^{-\lambda_i \cdot t} \right]$$

The form of the above equation is identified with a finite state, discrete time, Markov process of order one, with stationary transition probabilities.

Hence, the circuit process is Markovian, if process of each of the component parameters is also Markovian.

At this stage, a question arises. Does the Markovian relationship for the circuit hold good, if the circuit transition was from $(JSCT)_n$ to $(JSCT)_{n+1}$.

Let the circuit change from state 'i' at time 't' to state 'j' at time 't+τ.'

At time: t= t

$(JSR1)_0 = q$

$(JSR2)_0 = v$

$(JSR3)_0 = w$

$(JSCT)_n = i$

At time $t = t+\tau$:

$$(JSR1)_0 = q$$

$$(JSR2)_0 = v$$

$$(JSR3)_1 = 1$$

$$(JSCT)_{n+1} = j$$

Specifically, in order that parameter R2 continues to remain in state 'v' for a duration of time that is less than or equal to 'τ', the following condition must hold good:

$$\Pr\left\{T_{1(R2)} \leq t + \tau \;/\; (JSR2)_0 = v,\ T_{1(R2)} \geq t\right\}$$

$$= 1 - e^{-\lambda v \cdot \tau}$$

It is noticed that the probability that R2 continues to remain in state 'v' is independent of the time spent in the previous state, and is entirely dependent on the time difference 'τ'. Where a parameter takes less time than all other parameters to change state, it will effect the change in state of the circuit earlier. Thus a Markovian relationship does hold good for all circuit transitions, and for $i,j = 1, 2, \ldots, m$.

It will be assumed that the probability of change of state of the circuit, due to change of states of two or more parameters is negligible.

THE DERIVATION OF THE RELIABILITY EQUATION

Relationship between a continuous time Markov process and a discrete time Markov process can be expressed in the form,

$P(t), = e^{\Lambda \cdot t}$

Where,

$\underset{mxm}{P(t)}$ - A matrix, whose elements $P_{ij}(t)$ are defined as the probability that the circuit is in state 'j' at time 't', given that the circuit was in state 'i' at time 't'=0.

$\underset{mxm}{\Lambda}$ - An infinitesimal generator matrix whose elements λ_{ij} are so defined that $\lambda_{ij} \Delta t + 0 \cdot \Delta t$, i = j, is the probability that a circuit in state 'i' at time zero enters state 'j' by time Δt with $\lambda_{ij} = -\sum_{i=j} \lambda_{ij}$

The proof of this equation is as follows:

For a sufficiently small 't',

$\lim_{t \to 0^+} P(t) = I,$

where I is a unit matrix.

From Chapman-Kolmogorov equation for a discrete time Markov process,

$P(t) \cdot P(\Delta t) = P(t + \Delta t), \quad t, \Delta t \geq 0$

Taking log of both sides,

$\log P(t) + \log P(\Delta t) = \log P(t + \Delta t).$

The above equation will hold good, only if log P(t) is continuous over time 't.' This suggests that log P(t) can be expressed in a linear relation of the form,

$\log P(t) = A + \Lambda t$: A, Λ are mxm matrices.

But the initial conditions specify,

$P(0) = I$

$\log P(0) = \log I = 0 = A + B \cdot 0 = 0$

Hence $A = 0$

Thus, $\log P(t) = \Lambda t$

$$P(t) = e^{\Lambda t}.$$

Λ is the constant slope of the linear function $\log P(t)$. Thus, Λ is also a matrix whose elements λ_{ij} are so defined that $\lambda_{ij} \cdot \Delta t + 0 \cdot (\Delta t)$, $i \neq j$, is the probability that a circuit in state 'i' at time zero enters state 'j' by time Δt with $\lambda_{ij} = \sum_{i \neq j} \lambda_{ij}$.

Let,

S_1 - Set of success states

S_2 - Set of failure states

Λ - Infinitesimal generator matrix that includes both S_1 and S_2.

Λ_t - Infinitesimal generator matrix that includes S_1 but excludes S_2.

a_o^i - The initial distribution of the circuit that includes both S_1 and S_2.

$\bar{a}_o^i$ - The initial distribution of the circuit that includes S_1 but excludes S_2.

If now, it is known that the circuit is operative at time 't' = 0, that is, only the operative circuit states are selected for use, then, the distribution of the circuit states at time 't' is,

$$= a_o^i \cdot P(t) \qquad i \varepsilon S_1$$

$$= a_o^i \cdot \left\{ P_{ij}(t) \right\} \qquad i \varepsilon S_1$$

$$= a_o^i \cdot \left\{ e^{\Lambda \cdot t} \right\} \qquad i \varepsilon S_1$$

$$= a_o^i \cdot e^{\{\lambda ij\} \cdot t} \qquad i \varepsilon S_1$$

$$= \bar{a}_o^i \cdot e^{\Lambda_t \cdot t}$$

The expression for the distribution of the circuit at time 't' is defined for a particular state 'j' at time 't.' Summing over all $j \varepsilon s_1$, the probability of the circuit not failing before time 't' is obtained. This is the circuit reliability R(t), given by the expression:

$$R(t) = \sum_j \bar{a}_i^o \cdot e^{\Lambda_t \cdot t}$$

$$= \bar{a}_i^o \cdot e^{\Lambda_t \cdot t} . I$$

$$R_i(t) = e^{\Lambda_t \cdot t} . I$$

In the latter sections, systematic procedures will be developed for the partitioning into states of the component parameter, and for the approximate determination of the circuit failure boundaries.

ESTIMATION AND TESTING OF MARKOV CHAINS

Various assumptions have been made earlier, in the derivation of the reliability equation. These are:

(1) The transition probabilities of the finite state, discrete time, Markov process are stationary.

(2) The Markov chain is of first order.

It is the purpose of this section to develop statistical techniques for validating the above assumptions. In addition, the method of estimation of the transition probabilities of the Markov chain will be presented.

ESTIMATION OF THE TRANSITION PROBABILITIES OF A FIRST ORDER MARKOV CHAIN

Let,

n - The number of statistically identical electronic component parts put on test.

n_{ij} - The number of parameters observed in state 'i' at the $(t-1)^{st}$ period and observed in state 'j' at the t^{th} period.

$n_i(t-1)$ - The number of parameters observed in state 'i' at the $(t-1)^{st}$ period.

T - The period over which the test is conducted.

m - The number of states into which the range of data is partitioned.

$$n_i(t-1) = \sum_{i=j}^{m} n_{ij}(t)$$

THE RESULTS OF THE TEST
FOR THE 't^{th}' PERIOD

$i \backslash j$	1	2	3	4	..	j	..	m
1	$n_{11}(t)$	$n_{12}(t)$	$n_{13}(t)$	$n_{14}(t)$		$n_{1j}(t)$		$n_{1m}(t)$
2	$n_{21}(t)$	$n_{22}(t)$	$n_{23}(t)$	$n_{24}(t)$		$n_{2j}(t)$		$n_{2m}(t)$
3	$n_{31}(t)$	$n_{32}(t)$	$n_{33}(t)$	$n_{34}(t)$		$n_{3j}(t)$		$n_{3m}(t)$
4	$n_{41}(t)$	$n_{42}(t)$	$n_{43}(t)$	$n_{44}(t)$		$n_{4j}(t)$		$n_{4m}(t)$
:								
i	$n_{i1}(t)$	$n_{i2}(t)$	$n_{i3}(t)$	$n_{i4}(t)$		$n_{ij}(t)$		$n_{im}(t)$
:								
m	$n_{m1}(t)$	$n_{m2}(t)$	$n_{m3}(t)$	$n_{m4}(t)$		$n_{mj}(t)$		$n_{mm}(t)$

TRANSITION PROBABILITIES TO ESTIMATE
FOR THE 't^{th}' PERIOD

$i \backslash j$	1	2	3	4	..	j	..	m
1	$p_{11}(t)$	$p_{12}(t)$	$p_{13}(t)$	$p_{14}(t)$		$p_{1j}(t)$		$p_{1m}(t)$
2	$p_{21}(t)$	$p_{22}(t)$	$p_{23}(t)$	$p_{24}(t)$		$p_{2j}(t)$		$p_{2m}(t)$
3	$p_{31}(t)$	$p_{32}(t)$	$p_{33}(t)$	$p_{34}(t)$		$p_{3j}(t)$		$p_{3m}(t)$
4	$p_{41}(t)$	$p_{42}(t)$	$p_{43}(t)$	$p_{44}(t)$		$p_{4j}(t)$		$p_{4m}(t)$
:								
i	$p_{i1}(t)$	$p_{i2}(t)$	$p_{i3}(t)$	$p_{i4}(t)$		$p_{ij}(t)$		$p_{im}(t)$
:								
m	$p_{m1}(t)$	$p_{m2}(t)$	$p_{m3}(t)$	$p_{m4}(t)$		$p_{mj}(t)$		$p_{mm}(t)$

The probability of getting $n_{i1}(t)$, $n_{i2}(t),\ldots,n_{im}(t)$ in some specified order for the t^{th} period is,

$$= p_{i1}(t)^{n_{i1}(t)}\; p_{i2}(t)^{n_{i2}(t)}, \ldots, p_{im}(t)^{n_{im}(t)}$$

$$= \prod_{j=1}^{m} P_{ij}(t)^{n_{ij}(t)}$$

The probability of getting $n_{i1}(t)$, $n_{i2}(t),\ldots,$ $n_{im}(t)$ in some specified order over the test period 'T' is,

$$= \prod_{t=1}^{T} \prod_{j=1}^{m} p_{ij}(t)^{n_{ij}(t)}$$

It will be seen that the set of number $n_{ij}(t)$ forms a set of sufficient statistics for estimating $p_{ij}(t)$.

The likelihood function for estimating $p_{ij}(t)$ is,

$$L = \prod_{j=1}^{m} p_{ij}(t)^{n_{ij}(t)}$$

$$L^* = \log L = \sum_{j=1}^{m} n_{ij}(t) \log p_{ij}(t)$$

The estimator $p_{ij}(t)$ can be found by equating the first derivatives of L^* equal to zero and solving for the parameters.

$$L^* = n_{i1}(t) \log p_{i1}(t) + n_{i2}(t) \log p_{i2}(t) + \ldots + n_{im}(t) \log p_{im}(t)$$

$$= n_{i1}(t) \log p_{i1}(t) + n_{i2}(t) \log p_{i2}(t) + n_{im}(t) \log \left[1 - p_{i1}(t) -- p_{im-1}(t)\right]$$

$$\frac{\partial L^*}{\partial p_{i1}(t)} = n_{i1}(t) \cdot \frac{1}{p_{i1}(t)} + n_{im}(t) \cdot \frac{(-1)}{1 - p_{i1}(t) -- p_{im-1}(t)}$$

$$= n_{i1}(t) \cdot \frac{1}{p_{i1}(t)} - n_{im}(t) \cdot \frac{1}{1 - p_{i1}(t) -- p_{im-1}(t)}$$

$$= 0$$

$$\frac{\partial L^*}{\partial p_{i2}(t)} = n_{i2}(t) \cdot \frac{1}{p_{i2}(t)} - n_{im}(t) \cdot \frac{1}{1 - p_{i1}(t) - - p_{im-1}(t)}$$

$$= 0$$

$$\frac{\partial L^*}{\partial p_{im}(t)} = n_{im}(t) \cdot \frac{1}{p_{im}(t)} - n_{im}(t) \frac{1}{1 - p_{i1}(t) - - p_{im-1}(t)}$$

$$= 0$$

Substituting $p_{im} = 1 - p_{i1}(t) - - p_{im-1}(t)$ and rearranging the equations,

$$n_{i1}(t) = p_{i1}(t) \cdot \frac{n_{im}(t)}{p_{im}(t)}$$

$$n_{i2}(t) = p_{i2}(t) \cdot \frac{n_{im}(t)}{p_{im}(t)}$$

$$n_{im}(t) = p_{im}(t) \cdot \frac{n_{im}(t)}{p_{im}(t)} .$$

Summing the left and right sides separately,

$$\sum_{j=1}^{m} n_{ij}(t) = \frac{n_{im}(t)}{p_{im}(t)} \sum_{j=1}^{m} p_{ij}(t)$$

$$n_i(t-1) = \frac{n_{im}(t)}{p_{im}(t)}$$

$$\left[\text{because, } \sum_{j=1}^{m} n_{ij}(t) = n_i(t-1); \sum_{j=1}^{m} p_{ij}(t) = 1\right]$$

Thus,

$$p_{im}(t) = \frac{n_{im}(t)}{n_i(t-1)}$$

that is,

$$\hat{p}_{ij}(t) = \frac{n_{ij}(t)}{n_i(t-1)} \text{ for } j = 1, 2, \ldots, m$$

For stationary transition probabilities,

$$\hat{p}_{ij} = \frac{n_{ij}}{n_i} = \frac{\sum_{t=1}^{T} n_{ij}(t)}{\sum_{t=1}^{T} n_i(t-1)} = \frac{\sum_{t=1}^{T} n_{ij}(t)}{\sum_{t=0}^{T-1} n_i(t)}$$

For large sample sizes, there is a theorem[7] which states that for large sample size, $\hat{p}_{ij}$ is normally distributed with a mean p_{ij}. This fact will be used to test the properties of the estimator $\hat{p}_{ij}$.

It is necessary to show that the estimator $\hat{p}_{ij}$ conforms to all the properties of a maximum likelihood estimator, namely the estimator is:

1. Unbiased
2. Efficient
3. Consistent
4. Sufficient

1. p_{ij} is normally distributed with the true value of the mean p_{ij}, that is,

$$E\,(\hat{p}_{ij}) = p_{ij}$$

Hence, the estimator p_{ij} is unbiased.

2. $\hat{p}_{ij}$ has the smallest variance of the set of estimators which are normally distributed.

Let $(\hat{p}_{ij})_1$ be any other estimator which is normally distributed. It can be shown that,

$$\frac{Var(\hat{p}_{ij})}{Var(\hat{p}_{ij})_1} < 1$$

Thus, $\hat{p}_{ij}$ is the most efficient estimator.

3. p_{ij} is a consistent estimate of p_{ij}.

For $N \to \infty$, the estimate of $\hat{p}_{ij} \to p_{ij}$; that is,

$$\lim_{N \to \infty} \hat{p}_{ij} = p_{ij}$$

$$i,j = 1, 2, 3, \ldots, m$$

There exists a number 'c' such that

$$\mathop{\mathrm{Prob}}_{N \to \infty} \left\{ \hat{p}_{ij} - p_{ij} > \tau \right\} < \eta,$$

Where,

$c, \eta > 0.$

$\eta \to 0$, as $N \to \infty$

4. The estimator utilizes all the data relevant to the estimation of the transition probabilities. It is thus a sufficient statistic.

TESTS OF HYPOTHESES

TEST OF THE HYPOTHESIS THAT THE TRANSITION PROBABILITIES ARE STATIONARY

Denoting the null hypothesis by 'H_0' and the alternative hypothesis by 'H_1', the requirement of the test is stated below:

H_0: $p_{ij}(t) = p_{ij}$ for $t = 1, 2, \ldots, T$

H_1: $p_{ij}(t)$ is dependent on 't'.

The estimate for $p_{ij}(t)$ is given by,

$$\hat{p}_{ij}(t) = \frac{n_{ij}(t)}{n_i(t-1)}$$

The likelihood function to be maximized under the null hypothesis is given by,

$$L = \prod_{T} \prod_{i,j=1}^{m} \hat{p}_{ij}^{\,n_{ij}(t)}$$

The likelihood function to be maximized under the alternative hypothesis is given by,

$$L' = \prod_{T} \prod_{i,j=1}^{m} \hat{p}_{ij}(t)^{n_{ij}(t)}$$

The likelihood ratio criteria is then,

$$\lambda = \frac{L}{L'} = \prod_{T} \prod_{i,j=1}^{m} \hat{p}_{ij}^{\,n_{ij}(t)} \frac{1}{\hat{p}_{ij}(t)^{n_{ij}(t)}}$$

It is known [6] that -2 log λ is distributed as chi-square with $(r-1)\left[m(m-1)\right]$ degrees of freedom. For any given region of confidence $1-\alpha$ in order that the null hypothesis is true, the following condition must be satisfied,

$$-2 \log \lambda \leq \chi^2_{\alpha,\,(r-1)\left[m(m-1)\right]}$$

χ^2 for any desired confidence interval and degrees of freedom can be found from any printed table of the chi-square distribution.

TEST OF THE HYPOTHESIS THAT A MARKOV CHAIN IS OF FIRST ORDER

Consider testing the null hypothesis that the Markov chain is of first order against the alternative hypothesis that it is second order.

H_0: The stationary Markov chain is of first order against,

H_1: The stationary Markov chain is of second order.

The maximum likelihood estimate for stationary, second order transition probabilities is given by,

$$\hat{p}_{ijk} = \frac{n_{ijk}}{\sum_{l=1}^{m} n_{ijl}} = \frac{\sum_{t=2}^{T} n_{ijk}(t)}{\sum_{t=2}^{T} n_{ij}(t-1)}$$

The null hypothesis states that,

$$p_{ijk} = p_{2jk} = \dots = p_{mjk} = p_{jk}$$

for $j, k = 1, 2, \dots, m$.

The likelihood ratio criterion for testing the hypothesis is,

$$\bar{\lambda} = \frac{L(\text{under } H_0)}{L(\text{under } H_1)}$$

$$= \frac{\prod_{j,k=1}^{m} \hat{p}_{jk}^{\,n_{jk}}}{\prod_{i,j,k=1}^{m} \hat{p}_{ijk}^{\,n_{ijk}}}$$

$$= \prod_{i,j,k=1}^{m} \left(\frac{\hat{p}_{jk}}{\hat{p}_{ijk}}\right)^{n_{ijk}}$$

(for a given i,j)

From the knowledge of the fact that $-2 \log \bar{\lambda}$ is distributed as χ^2 for a given region of confidence $1-\alpha$ with $m(m-1)$ degrees of freedom, it is possible to state the condition for H_0 to be true.

If H_0 is true, then,

$$-2 \log \bar{\lambda} \leq \chi^2_{\alpha, m(m-1)^2}$$

$\chi^2_{\alpha, m(m-1)^2}$ can be found from any printed table of the chi-square distribution.

A GENERALIZED MARKOVIAN RELIABILITY
ANALYSIS OF AN 'r' ELEMENT ELECTRONIC CIRCUIT

DATA

Let 'N' statistically identical electronic component parts be put on test over a period 'T.' If there are 'r' different component parameters denoted by Z1, Z2,..., and Zr respectively. Then the ranges of each of these parameters as obtained from the test can be summarized as below:

$Z_b^1 \leq Z1 \leq Z_a^1$ - Parameter Z1

$Z_b^2 \leq Z2 \leq Z_a^2$ - Parameter Z2

. . . . . .

$Z_b^r \leq Zr \leq Z_a^r$ - Parameter Zr

Let each of these ranges be partitioned into disjoint intervals m_1, m_2,..., and m_r states respectively. By definition, each such interval will be a state.

State - 1 $Z_b^1 \leq Z1 < Z_1^1$

State - 2 $Z_1^1 \leq Z1 < Z_2^1$ Parameter Z1

.
.
.

State - m_1 $Z_{m-1}^1 \leq Z1 \leq Z_a^1$

State - 1 $Z_b^2 \leq Z2 < Z_1^2$

State - 2 $Z_1^2 \leq Z2 < Z_2^2$ Parameter Z2

.
.
.

State - m_2 $Z_{m_2-1}^2 \leq Z2 \leq Z_a^2$

.
.
.

State - 1 $\quad Z_b^r \leq Zr < Z_1^r$

State - 2 $\quad Z_1^r \leq Zr < Z_2^r$ $\qquad$ Parameter Zr

.
.
.

State - m_r $\quad Z_{m_r-1}^r \leq Zr \leq Z_a^r$

Let $n_{ij}(t)$ be the number of parameters observed in state 'i' at the $(t-1)^{st}$ period and observed in state 'j' at the t^{th} period, for any given component part.

The number of parameters observed in state 'i' at the $(t-1)^{st}$ period, for the given component part, is then

$$n_i(t-1) = \sum_j n_{ij}(t)$$

$$N = \sum_i n_i(t-1) = \sum_i \sum_j n_{ij}(t)$$

The transition probabilities $\{p_{ij}\}$ are estimated from the data, for each of the component parameters. Testing procedures, outlined earlier, are then applied to verify that the data came from a stationary, first order, Markov process.

The state probabilities for the circuit can be found as below.

Let the initial state 'i' of the circuit be expressed as,

$$i = (l^1, m^1, \ldots, q^1)$$

'r' component parameters

The probability that the circuit remains in state 'i' is,

$$a_i^o = P_r\left\{ JSZ1 = l^1 \right\} \cdot P_r\left\{ JSZ2 = m^1 \right\} \cdot \cdot \cdot P_r\left\{ JSZr = q^1 \right\}$$

Component parameters Z1, Z2,..., and Zr are subject to an initial probability distribution as,

$$P_r\left\{ JSZj = l \right\} = \frac{n_l(0)}{N}$$

where $j = 1, 2, \ldots, r$

$l = 1, 2, \ldots, m_1$ for $j = 1$

$= 1, 2, \ldots, m_2$ for $j = 2$

. . . . .

$= 1, 2, \ldots, m_r$ for $j = r$

DERIVATION OF THE CIRCUIT STATES FROM THE COMPONENT STATES

Let the circuit be considered operable, if the output parameter V_{OUT} is subject to the condition,

$$b \leq V_{OUT} \leq a$$

By circuit analysis it is possible to establish the functional relationship between V_{OUT} and the various component parameters.

$$V_{OUT} = f(Z1, Z2, \ldots, Zr)$$

Next, the relationship between the circuit state and the component parameter states will be derived. Total number of circuit states possible $= m_1 \times m_2 \times, \ldots, m_r$

$$= \prod_{i=1}^{r} m_i$$

Let JSCT be the circuit state number, JSCT = 1, 2,...,

$$\prod_{i=1}^{r} m_i$$

Let percentage change in Z1 > percentage change in Z2 > percentage change in Z3 and so on. Then,

$$JSCT = \prod_{i=2}^{r} m_i \quad (JSZ1-1) + \prod_{i=3}^{r} m_i \quad (JSZ2-1) + \ldots$$

$$+ m_r \quad (JSZ(r-1)-1) + JSZr$$

This equation is obvious from the fact that for each state of Z(r-1) there are m_r states of Zr, for each state of Z(r-2) there are $m_{r-1} \times m_r$ states of Z(r-1), and so on.

For example, let

JSZ1 = 1, 2

JSZ2 = 1, 2

JSZ3 = 1, 2

Number of circuit states possible = 2x2x2=8.

If the percentage change in Z1 $<$ the percentage change in Z2 $<$ the percentage change in Z3, then

JSCT = 4 (JSZ1 -1) + 2(JSZ2 -1) + JSZ3

JSZ1	JSZ2	JSZ3	JSCT
1	1	1	1
1	1	2	2
1	2	1	3
1	2	2	4
2	1	1	5
2	1	2	6
2	2	1	7
2	2	2	8

The scheme, shown above, summarizes the circuit states in terms of the component parameter states.

DETERMINATION OF THE FAILURE BOUNDARIES OF THE CIRCUIT

In order to know whether a particular state 'JSCT' is a success or a failure, it is necessary to obtain the value of V_{OUT} corresponding to the values of Z1, Z2,..., and Zr, of the states, JSZ1, JSZ2,..., and JSZr respectively. If the value of V_{OUT} lies within the operable range then JSCT = (JSZ1, JSZ2,...,JSZr) yields a state of success.

A computer program can be developed to compute the values of V_{OUT} for various values of Z1, Z2,..., and Zr. (States of Z1, Z2,..., and Zr.)

The corresponding state number of the circuit and the circuit condition success or failure can be easily recorded. Often, a circuit state may be found to lie in the regions of both success and failure. By redefining the states of the component parameters, it is possible to obtain a result from which accurate failure boundaries can be obtained.

Let the range of values for a component parameter Zi be partitioned into m_i states as below:

State - 1	$Z^i_b \leq Zi < Z^i_1$	
State - 2	$Z^i_1 \leq Zi < Z^i_2$	Parameter Zi
.		
.		
.		
State - j	$Z^i_{j-1} \leq Zi < Z^i_j$	
.		
.		
.		
State - m_i	$Z^i_{m_{i-1}} \leq Zi \leq Z^i_a$	

State -1	State -2	. . .	State -j	. .	State -m_i
Z^i_b Z^i_1	Z^i_2		Z^i_{j-1} Z^i_j		$Z^1_{m_{i-1}}$ Z^1_a

Each interval is divided into several sub-intervals. The length of each of these sub-intervals is made as small as possible, within the limitation of the computing capacity.

The value of each of the parameters is incremented by a value of the sub-interval length. Knowing further, the relationship between JSCT and JSZi (i = 1, 2,...,r) for all 'i', the computer is programmed to record the following values at the output:

$$Zi \quad Z2 \cdots Zr \quad V_{OUT} \quad JSZ1 \quad JSZ2 \cdots JSZr \quad JSCT \quad K$$

'K' further signifies whether the circuit is in a state of success (1) or in a state of failure (0).

Let N_F be the number of failure states of the circuit as obtained from the computer output.

SYNTHESISATION OF THE CIRCUIT MATRIX FROM THE COMPONENT MATRICES

If

p_{Zi} - The finite state, discrete time, transition probability matrix of the parameter Zi.

Λ_{Zi} - The finite state, continuous time, transition probability matrix.

Then,

$p_{Zi} = e^{\Lambda_{Zi}}$ (The discrete time process data are obtained in steps of time interval, t=1.)

From the above relation,

$$\log p_{Zi} = \Lambda_{Zi}$$

Let $Q_{Zi} = p_{Zi} - I$

$$\Lambda_{Zi} = \log p_{Zi} = \log (Q_{Zi} + I)$$

$$= Q_{Zi} - \frac{Q_{Zi}^2}{2} + \frac{Q_{Zi}^3}{3} - \frac{Q_{Zi}^4}{4} + $$

This series in Q_{Zi} will converge to $\log p_{Zi}$ only when the diagonal elements of $p_{Zi} \geq \frac{1}{2}$.

It is possible, at this stage, that the matrix λ_{Zi} has yielded some negative off diagonal elements. This may be due to some sample variations indicating that the sample did not come from a continuous time process. There are two ways of handling this situation: λ_{Zi} may be either altered to eliminate the negative off diagonal elements by suitably distributing this effect, or alternatively work with the same matrix. The second course would, however, be chosen as this would cause less cumulative error.

Having obtained Λ_{Zi} matrices for all 'i' (that is, all the parameters), it is next necessary to develop procedures for synthesing the circuit matrix Λ_C from the component parameter matrices.

For the sake of illustration, consider a circuit consisting of three parameters Z1, Z2, Z3. Let the equation, relating the circuit state number to the component state numbers be given by,

$$JSCT = 4(JSZ1) + 2(JSZ2 - 1) + JSZ3.$$

Let,

$$\Lambda_{Z1} = \begin{bmatrix} \lambda_{11}(Z1) & \lambda_{12}(Z1) \\ \lambda_{21}(Z1) & \lambda_{22}(Z1) \end{bmatrix}$$

$$\Lambda_{Z2} = \begin{bmatrix} \lambda_{11}(Z2) & \lambda_{12}(Z2) \\ \lambda_{21}(Z2) & \lambda_{22}(Z2) \end{bmatrix}$$

$$\Lambda_{Z3} = \begin{bmatrix} \lambda_{11}(Z3) & \lambda_{12}(Z3) \\ \lambda_{21}(Z3) & \lambda_{22}(Z3) \end{bmatrix}$$

From the equation relating JSCT, JSZ1, JSZ2, and JSZ3, the following scheme can be derived.

JSZ1	JSZ2	JSZ3	JSCT
1	1	1	1
1	1	2	2
1	2	1	3
1	2	2	4
2	1	1	5
2	1	2	6
2	2	1	7
2	2	2	8

The circuit output parameter matrix will be of the form,

$$\begin{array}{c} \\ 1 \\ 2 \\ \vdots \\ 8 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & \cdots & 8 \end{array} \\ \begin{bmatrix} \lambda_{11}(C) & \lambda_{12}(C) & & \lambda_{18}(C) \\ \lambda_{21}(C) & \lambda_{22}(C) & & \lambda_{28}(C) \\ & & & \\ \lambda_{81}(C) & \lambda_{82}(C) & & \lambda_{88}(C) \end{bmatrix} \end{array}$$

A Markov process was derived for the circuit in an earlier section. The circuit was found to follow a Markov process, only when the transitions of the circuit states occurred due to change of state of one component parameters. It will, therefore, be assumed that the probability of circuit transition, due to change of states of more than one component parameter, is zero. It follows that the probability of circuit transition = probability of the component parameter transition, effecting the circuit transition.

With these ground rules, it is possible to derive the transition probability matrix for the circuit.

To calculate the elements of the first row of the circuit matrix, the following table is derived.

Circuit 'C'		Parameter Z1		Parameter Z2		Parameter Z3	
Present State	Next State	Present State	Next State	Present State	Next State	Present State	Next State
1	1	1	1	1	1	1	1
1	2	1	1	1	1	1	2
1	3	1	1	1	2	1	1
1	4	1	1	1	2	1	2
1	5	1	2	1	1	1	1
1	6	1	2	1	1	1	2
1	7	1	2	1	2	1	1
1	8	1	2	1	2	1	2

(1) $\lambda_{11(C)} = -\sum_{j=2}^{8} \lambda_{ij}$

It is necessary to know λ_{12}, λ_{13},..., and λ_{18}, in order to calculate λ_{11}.

(2) $\lambda_{12(C)} = \lambda_{12(Z3)}$, since the change in circuit state from state -1 to state -2 was brought about by the change of state of the parameter Z3 from state -1 to state -2.

(3) $\lambda_{13(C)} = \lambda_{12(Z2)}$, since the change in circuit state was effected by the change of state of the parameter Z2 only.

(4) $\lambda_{14(C)} = 0$, as the probability of the circuit transition due to simultaneous change of states of parameters Z2 and Z3 is zero.

(5) $\lambda_{15(C)} = \lambda_{12(Z1)}$, change of state of the circuit being effected, only by the change of state of Z1.

(6) $\lambda_{16(C)} = 0$, due to simultaneous changes of states of Z1 and Z3.

(7) $\lambda_{17(C)} = 0$, due to simultaneous changes of states of Z1 and Z2.

(8) $\lambda_{18(C)} = 0$, due to simultaneous changes of states Z1, Z2, and Z3.

Thus,

$$\begin{aligned}\lambda_{11(C)} &= -\left\{ \lambda_{12(C)} + \lambda_{13(C)} + \lambda_{15(C)} + \lambda_{16(C)} + \lambda_{17(C)} + \lambda_{18(C)} \right\} \\ &= -\left\{ \lambda_{12(Z3)} + \lambda_{12(Z2)} + \lambda_{12(Z1)} \right\}.\end{aligned}$$

Proceeding as before, all the other row vectors of Λ_C can be determined. Hence, Λ_C can be derived for an 'r' element circuit.

COMPUTATION OF THE RELIABILITY OF THE CIRCUIT

Having obtained the necessary ingredients to compute reliability, the reliability at various time points can be calculated.

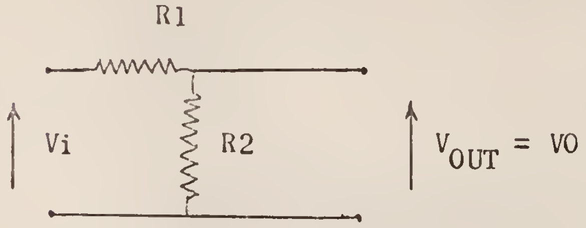
Cancelling those states of n_f which correspond to failure from the initial distribution vector a_0^i and the infinitesimal generator matrix Λ_C of the circuit, new matrices $\bar{a}_0^i$ and Λ_t are obtained. Hence,

$$R(t) = \bar{a}_0^i \cdot e^{\Lambda_t \cdot t} \cdot I$$

$$= \bar{a}_0^i \left[I + \Lambda_t \cdot I \cdot t + (\Lambda_t^2 \cdot I) \cdot \frac{t^2}{2!} + (\Lambda_t^3 \cdot I) \frac{t^3}{3!} + \ldots \right]$$

APPLICATION OF THE MARKOVIAN RELIABILITY ANALYSIS TO A SIMPLE TWO RESISTOR CIRCUIT

The following hypothetical circuit is assumed.



By circuit analysis,

$$Vi = IR1 + IR2,$$

$$VO = IR2.$$

$$\frac{VO}{Vi} = \frac{R2}{R1 + R2}$$

Hence, $VO = \frac{R2}{R1 + R2} \cdot Vi$

The circuit will be considered operable under the condition:

$$0.1997 \text{ V} \leq VO \leq 0.2130 \text{ V}$$

Let the nominal values of the circuit parameters be

$R1 = 5200\ \Omega$

$R2 = 1400\ \Omega$

Let the life test data yield the following ranges of values for the resistors:

R1 : $5000\ \Omega - 5750\ \Omega$

R2 : $1300\ \Omega - 1600\ \Omega$

Suppose, that these ranges of R1 and R2 can be partitioned into the following states:

For R1:

State: 1 $R1 \leq 5500\ \Omega$

State: 2 $R1 > 5500\ \Omega$

For R2:

State: 1 $R2 \leq 1400\ \Omega$

State: 2 $1400 < R2 < 1540$

State: 3 $R2 \geq 1540$

It is noticed that the parameter R2 has a larger percentage variation than parameter R1.

$$\text{Percentage variation of R1} = \frac{5.75 - 5.0}{5.0} \times 100 = 15\%$$

$$\text{Percentage variation of R2} = \frac{1600 - 1300}{1300} \times 100 = 25.8\%$$

Hence the circuit state will be adequately represented by the equation,

$$JSCT = 3(JSR1 - 1) + JSR2$$

The following table can be derived from this equation

JSR1	JSR2	JSCT
1	1	1
1	2	2
1	3	3
2	1	4
2	2	5
2	3	6

Due to lack of actual test data, the following first order, stationary, transition probability matrices for the component parameters R1 and R2 are assumed:

$$P_{R1} = \begin{bmatrix} 0.9889 & 0.0111 \\ 0.0191 & 0.9809 \end{bmatrix}$$

$$P_{R2} = \begin{bmatrix} 1.0 & 0 & 0 \\ 0.007 & 0.9745 & 0.0186 \\ 0 & 0.0033 & 0.9967 \end{bmatrix}$$

Since $p_{ii} > \frac{1}{2}$ for each of the above matrices, the equivalent continuous time matrices can be written as,

$$\Lambda_{R1} = \log P_{R1} = \log (Q_{R1} + I) = Q_{R1} - \frac{Q_{R1}^2}{2} + \frac{Q_{R1}^3}{3} + \ldots$$

$$\Lambda_{R2} = \log P_{R2} = \log (Q_{R2} + I)$$

$$= Q_{R2} - \frac{Q_{R2}^2}{2} + \frac{Q_{R2}^3}{3} + \ldots$$

Thus,

$$Q_{R1} = \begin{bmatrix} 0.9889 & 0.0111 \\ 0.0191 & 0.9809 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -0.0111 & 0.0111 \\ 0.0191 & -0.0191 \end{bmatrix}$$

$$Q_{R2} = \begin{bmatrix} 1.0 & 0 & 0 \\ 0.007 & 0.9745 & 0.0186 \\ 0 & 0.0033 & 0.9967 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0.0070 & 0.0255 & 0.0186 \\ 0 & 0.0033 & 0.0033 \end{bmatrix}$$

Hence,

$$\Lambda_{R1} = \begin{bmatrix} -0.0111 & 0.0111 \\ -0.0191 & -0.0191 \end{bmatrix}$$

$$\Lambda_{R2} = \begin{bmatrix} 0 & 0 & 0 \\ 0.0071 & -0.0259 & 0.0189 \\ 0 & 0.0033 & -0.0033 \end{bmatrix}$$

The computer is programmed such that for incremental values of 50Ω and 20Ω for R1 and R2 respectively, the following information is printed out at the output.

value of R1 value of R2 value of VO JSR1 JSR2 JSCT K

Circuit states 1, 3, 5 and 6 are designated as failure, as these states have been found to fail most often.

Deleting those states corresponding to failure from the Λ_C matrix, the resultant circuit matrix takes the form:

	1	2	3	4	5	6
1						
2		λ_{22}		λ_{24}		
3						
4		λ_{42}		λ_{44}		
5						
6						

That is,

$$\begin{array}{cc} & \begin{array}{cc} 2 & 4 \end{array} \\ \begin{array}{c} 2 \\ 4 \end{array} & \begin{bmatrix} \lambda_{22} & \lambda_{24} \\ \lambda_{42} & \lambda_{44} \end{bmatrix} \end{array}$$

The following table gives the circuit state transitions in terms of the transitions of the component parameter states.

Circuit 'C'		Parameter R1		Parameter R2	
Present State	Next State	Present State	Next State	Present State	Next State
2	1	1	1	2	1
2	2	1	1	2	2
2	3	1	1	2	3
2	4	1	2	2	1
2	5	1	2	2	2
2	6	1	2	2	3
4	1	2	1	1	1
4	2	2	1	1	2
4	3	2	1	1	3
4	4	2	2	1	1

Table (Continued)

Circuit 'C'		Parameter R1		Parameter R2	
Present State	Next State	Present State	Next State	Present State	Next State
4	5	2	2	1	2
4	6	2	2	1	3

Thus,

$$\lambda_{21(C)} = \lambda_{21(R2)} = 0.0071$$

$$\lambda_{22(C)} = -\left[\lambda_{21(C)} + \lambda_{23(C)} + \lambda_{24(C)} + \lambda_{25(C)} + \lambda_{26(C)} \right]$$

$$\lambda_{23(C)} = \lambda_{23(R2)} = 0.0189$$

$$\lambda_{24(C)} = 0$$

$$\lambda_{25(C)} = \lambda_{12(R1)} = 0.0111$$

$$\lambda_{26(C)} = 0$$

Hence,

$$\lambda_{22(C)} = -\left[0.0071 + 0.0189 + 0.0111 \right]$$

$$= -0.0371$$

$$\lambda_{24(C)} = 0$$

Now,

$$\lambda_{41(C)} = \lambda_{21(R1)} = 0.0191$$

$$\lambda_{42(C)} = 0$$

$$\lambda_{43(C)} = 0$$

$$\lambda_{44(C)} = -(\lambda_{41} + \lambda_{42} + \lambda_{43} + \lambda_{45} + \lambda_{46})$$

$$\lambda_{45(C)} = -\lambda_{12(R2)} = 0$$

$$\lambda_{46(C)} = \lambda_{13(R2)} = 0$$

$$\therefore \lambda_{44(C)} = -0.0191$$

$$\lambda_{42(C)} = 0$$

Hence, the infinitesimal generator matrix for the circuit is,

$$\Lambda_t = \begin{bmatrix} \lambda_{22}(C) & \lambda_{24}(C) \\ \lambda_{42}(C) & \lambda_{44}(C) \end{bmatrix} = \begin{bmatrix} -0.0371 & 0 \\ 0 & -0.0191 \end{bmatrix}$$

$$R_i(t) = I + \Lambda_t \cdot I \cdot t + (\Lambda_t^2 \cdot I) \cdot \frac{t2}{21} + (\Lambda_t^3 \cdot I)\ \frac{t^3}{31}\ \ldots,$$

$$\Lambda_t = \begin{bmatrix} -0.0371 & 0 \\ 0 & -0.0191 \end{bmatrix}$$

$$\Lambda_t^2 = \begin{bmatrix} -0.0371 & 0 \\ 0 & -0.0191 \end{bmatrix} \cdot \begin{bmatrix} -0.0371 & 0 \\ 0 & -0.0191 \end{bmatrix}$$

$$= \begin{bmatrix} 0.001376 & 0 \\ 0 & 0.000364 \end{bmatrix}$$

Neglecting the higher powers of Λ_t^3, Λ_t^4 ..., etc.,

$$R_i(t) = \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} -0.0371 & 0 \\ 0 & -0.0191 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix} \cdot t + \begin{bmatrix} 0.001376 & 0 \\ 0 & 0.000364 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix} \cdot \frac{t^2}{21} +$$

$$\therefore \quad R_i(t) = \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} -0.0371 \\ -0.0191 \end{bmatrix} \cdot t + \begin{bmatrix} 0.001376 \\ 0.000364 \end{bmatrix} \cdot \frac{t^2}{2}$$

Let the initial probability vector for the circuit, as estimated from the knowledge of life test data be,

$$(0,\ \tfrac{1}{2},\ 0,\ \tfrac{1}{2},\ 0,\ 0)$$

Deleting those states that represent failure, the initial probability vector is then,

$$(\tfrac{1}{2},\tfrac{1}{2})$$

If now t is in units of months,

$$R\ (1) = (\tfrac{1}{2},\tfrac{1}{2}) \cdot R_i(1)$$

$$= (\tfrac{1}{2},\tfrac{1}{2}) \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} -0.0371 \\ 0.0191 \end{bmatrix} \cdot 1 + \begin{bmatrix} 0.001376 \\ 0.000364 \end{bmatrix} \cdot \frac{1}{2} \right\}$$

$$= (\tfrac{1}{2},\tfrac{1}{2}) \cdot \begin{bmatrix} 0.9642 \\ 0.9812 \end{bmatrix}$$

$$= \tfrac{1}{2} \times 1.9454 = \underline{0.9727}$$

$$R(2) = (\tfrac{1}{2}, \tfrac{1}{2})\left\{\begin{bmatrix}1\\1\end{bmatrix} + \begin{bmatrix}-0.0371\\0.0191\end{bmatrix} \cdot 2 + \begin{bmatrix}0.001376\\0.000364\end{bmatrix} \cdot \frac{2^2}{2}\right\}$$

$$= (\tfrac{1}{2}, \tfrac{1}{2}) \cdot \begin{bmatrix}0.9285\\0.9625\end{bmatrix}$$

$$= \tfrac{1}{2} \times 1.8910 = 0.9455$$

$$R(3) = (\tfrac{1}{2}, \tfrac{1}{2})\left\{\begin{bmatrix}1\\1\end{bmatrix} + \begin{bmatrix}-0.0371\\-0.0191\end{bmatrix} \cdot 3 + \begin{bmatrix}0.001376\\0.000364\end{bmatrix} \cdot \frac{3^2}{2}\right\}$$

$$= (\tfrac{1}{2}, \tfrac{1}{2}) \cdot \begin{bmatrix}0.8950\\0.9443\end{bmatrix}$$

$$= \tfrac{1}{2} \times 1.8393 = \underline{0.9196}$$

$$R(4) = (\tfrac{1}{2}, \tfrac{1}{2})\left\{\begin{bmatrix}1\\1\end{bmatrix} + \begin{bmatrix}-0.0371\\-0.0191\end{bmatrix} \cdot 4 + \begin{bmatrix}0.001376\\0.000364\end{bmatrix} \cdot \frac{4^2}{4}\right\}.$$

$$= (\tfrac{1}{2}, \tfrac{1}{2}) \cdot \begin{bmatrix}0.8568\\0.9248\end{bmatrix}$$

$$= \tfrac{1}{2} \times 1.7816 = \underline{0.8908}$$

Thus,

$R(1) = \underline{0.9727}$

$R(2) = \underline{0.9455}$

$R(3) = \underline{0.9196}$

$R(4) = \underline{0.8908}$

```
00010 FORMAT(2X,2I5)
00140 FORMAT(2X,2I5,F6.4,4I5)
00005 FORMAT(4X,2HR1,3X,2HR2,3X,2HVO,5X,17H JSR1 JSR2 JSCT K)
      READ(1,10)I,J
      WRITE(3,5)
00006 VI=1.
      DO135I=5000,5750,50
      R1=I+50
      DO135J=1300,1600,20
      R2=J+20
      VO=(R2/(R1+R2))*VI
00035 IF(5500.-R1)40,45,45
00040 JSR1=1
      GOTO50
00045 JSR1=2
00050 IF(1400.-R2)51,55,55
00051 IF(1540.-R2)52,52,60
00055 JSR2=1
      GOTO145
00060 JSR2=2
      GOTO145
00052 JSR2=3
00145 IF(JSR1-1)135,70,65
00070 IF(JSR2-1)135,90,82
00065 IF(JSR1-2)135,83,135
00082 IF(JSR2-2)135,95,84
00083 IF(JSR2-1)135,105,85
00084 IF(JSR2-3)135,100,135
00085 IF(JSR2-2)135,106,86
00086 IF(JSR2-3)135,107,135
00090 JSCT=1
      GOTO110
00095 JSCT=2
      GOTO110
00100 JSCT=3
      GOTO110
00105 JSCT=4
      GOTO110
00106 JSCT=5
      GOTO110
00107 JSCT=6
00110 IF(0.1997-VO)115,130,125
00115 IF(VO-0.2130)130,130,125
00125 K=0
      GOTO138
00130 K=1
00138 WRITE(3,140)I,J,VO,JSR1,JSR2,JSCT,K
00135 CONTINUE
      END
```

R1	R2	VO	JSR1	JSR2	JSCT	K
5000	1300	.2072	2	1	4	1
5000	1320	.2097	2	1	4	1
5000	1340	.2121	2	1	4	1
5000	1360	.2146	2	1	4	0
5000	1380	.2170	2	1	4	0
5000	1400	.2194	2	2	5	0
5000	1420	.2218	2	2	5	0
5000	1440	.2242	2	2	5	0
5000	1460	.2266	2	2	5	0
5000	1480	.2290	2	2	5	0
5000	1500	.2313	2	2	5	0
5000	1520	.2336	2	3	6	0
5000	1540	.2360	2	3	6	0
5000	1560	.2383	2	3	6	0
5000	1580	.2406	2	3	6	0
5000	1600	.2428	2	3	6	0
5050	1300	.2056	2	1	4	1
5050	1320	.2080	2	1	4	1
5050	1340	.2105	2	1	4	1
5050	1360	.2129	2	1	4	1
5050	1380	.2153	2	1	4	0
5050	1400	.2177	2	2	5	0
5050	1420	.2201	2	2	5	0
5050	1440	.2225	2	2	5	0
5050	1460	.2249	2	2	5	0
5050	1480	.2272	2	2	5	0
5050	1500	.2296	2	2	5	0
5050	1520	.2319	2	3	6	0
5050	1540	.2342	2	3	6	0
5050	1560	.2365	2	3	6	0
5050	1580	.2388	2	3	6	0
5050	1600	.2410	2	3	6	0
5100	1300	.2040	2	1	4	1
5100	1320	.2064	2	1	4	1
5100	1340	.2089	2	1	4	1
5100	1360	.2113	2	1	4	1
5100	1380	.2137	2	1	4	0
5100	1400	.2161	2	2	5	0
5100	1420	.2185	2	2	5	0
5100	1440	.2208	2	2	5	0
5100	1460	.2232	2	2	5	0
5100	1480	.2255	2	2	5	0
5100	1500	.2278	2	2	5	0
5100	1520	.2301	2	3	6	0
5100	1540	.2324	2	3	6	0
5100	1560	.2347	2	3	6	0
5100	1580	.2370	2	3	6	0
5100	1600	.2392	2	3	6	0
5150	1300	.2024	2	1	4	1
5150	1320	.2048	2	1	4	1
5150	1340	.2073	2	1	4	1
5150	1360	.2097	2	1	4	1
5150	1380	.2121	2	1	4	1
5150	1400	.2145	2	2	5	0

5150	1420	.2168	2	2	5	0
5150	1440	.2192	2	2	5	0
5150	1460	.2215	2	2	5	0
5150	1480	.2238	2	2	5	0
5150	1500	.2261	2	2	5	0
5150	1520	.2284	2	3	6	0
5150	1540	.2307	2	3	6	0
5150	1560	.2330	2	3	6	0
5150	1580	.2352	2	3	6	0
5150	1600	.2375	2	3	6	0
5200	1300	.2009	2	1	4	1
5200	1320	.2033	2	1	4	1
5200	1340	.2057	2	1	4	1
5200	1360	.2081	2	1	4	1
5200	1380	.2105	2	1	4	1
5200	1400	.2128	2	2	5	1
5200	1420	.2152	2	2	5	0
5200	1440	.2175	2	2	5	0
5200	1460	.2199	2	2	5	0
5200	1480	.2222	2	2	5	0
5200	1500	.2245	2	2	5	0
5200	1520	.2268	2	3	6	0
5200	1540	.2290	2	3	6	0
5200	1560	.2313	2	3	6	0
5200	1580	.2335	2	3	6	0
5200	1600	.2358	2	3	6	0
5250	1300	.1993	2	1	4	0
5250	1320	.2018	2	1	4	1
5250	1340	.2042	2	1	4	1
5250	1360	.2065	2	1	4	1
5250	1380	.2089	2	1	4	1
5250	1400	.2113	2	2	5	1
5250	1420	.2136	2	2	5	0
5250	1440	.2159	2	2	5	0
5250	1460	.2182	2	2	5	0
5250	1480	.2205	2	2	5	0
5250	1500	.2228	2	2	5	0
5250	1520	.2251	2	3	6	0
5250	1540	.2274	2	3	6	0
5250	1560	.2296	2	3	6	0
5250	1580	.2318	2	3	6	0
5250	1600	.2341	2	3	6	0
5300	1300	.1979	2	1	4	0
5300	1320	.2002	2	1	4	1
5300	1340	.2026	2	1	4	1
5300	1360	.2050	2	1	4	1
5300	1380	.2074	2	1	4	1
5300	1400	.2097	2	2	5	1
5300	1420	.2120	2	2	5	1
5300	1440	.2143	2	2	5	0
5300	1460	.2166	2	2	5	0
5300	1480	.2189	2	2	5	0
5300	1500	.2212	2	2	5	0
5300	1520	.2235	2	3	6	0
5300	1540	.2257	2	3	6	0

5300	1560	.2279	2	3	6	0
5300	1580	.2302	2	3	6	0
5300	1600	.2324	2	3	6	0
5350	1300	.1964	2	1	4	0
5350	1320	.1988	2	1	4	0
5350	1340	.2011	2	1	4	1
5350	1360	.2035	2	1	4	1
5350	1380	.2058	2	1	4	1
5350	1400	.2082	2	2	5	1
5350	1420	.2105	2	2	5	1
5350	1440	.2128	2	2	5	1
5350	1460	.2151	2	2	5	0
5350	1480	.2173	2	2	5	0
5350	1500	.2196	2	2	5	0
5350	1520	.2219	2	3	6	0
5350	1540	.2241	2	3	6	0
5350	1560	.2263	2	3	6	0
5350	1580	.2285	2	3	6	0
5350	1600	.2307	2	3	6	0
5400	1300	.1949	2	1	4	0
5400	1320	.1973	2	1	4	0
5400	1340	.1997	2	1	4	1
5400	1360	.2020	2	1	4	1
5400	1380	.2043	2	1	4	1
5400	1400	.2066	2	2	5	1
5400	1420	.2089	2	2	5	1
5400	1440	.2112	2	2	5	1
5400	1460	.2135	2	2	5	0
5400	1480	.2158	2	2	5	0
5400	1500	.2180	2	2	5	0
5400	1520	.2203	2	3	6	0
5400	1540	.2225	2	3	6	0
5400	1560	.2247	2	3	6	0
5400	1580	.2269	2	3	6	0
5400	1600	.2291	2	3	6	0
5450	1300	.1935	2	1	4	0
5450	1320	.1959	2	1	4	0
5450	1340	.1982	2	1	4	0
5450	1360	.2005	2	1	4	1
5450	1380	.2028	2	1	4	1
5450	1400	.2052	2	2	5	1
5450	1420	.2074	2	2	5	1
5450	1440	.2097	2	2	5	1
5450	1460	.2120	2	2	5	1
5450	1480	.2142	2	2	5	0
5450	1500	.2165	2	2	5	0
5450	1520	.2187	2	3	6	0
5450	1540	.2209	2	3	6	0
5450	1560	.2231	2	3	6	0
5450	1580	.2253	2	3	6	0
5450	1600	.2275	2	3	6	0
5500	1300	.1921	1	1	1	0
5500	1320	.1944	1	1	1	0
5500	1340	.1968	1	1	1	0
5500	1360	.1991	1	1	1	0

5500	1380	.2014	1	1	1	1
5500	1400	.2037	1	2	2	1
5500	1420	.2060	1	2	2	1
5500	1440	.2082	1	2	2	1
5500	1460	.2105	1	2	2	1
5500	1480	.2127	1	2	2	1
5500	1500	.2149	1	2	2	0
5500	1520	.2172	1	3	3	0
5500	1540	.2194	1	3	3	0
5500	1560	.2215	1	3	3	0
5500	1580	.2237	1	3	3	0
5500	1600	.2259	1	3	3	0
5550	1300	.1907	1	1	1	0
5550	1320	.1930	1	1	1	0
5550	1340	.1954	1	1	1	0
5550	1360	.1977	1	1	1	0
5550	1380	.2000	1	1	1	1
5550	1400	.2022	1	2	2	1
5550	1420	.2045	1	2	2	1
5550	1440	.2067	1	2	2	1
5550	1460	.2090	1	2	2	1
5550	1480	.2112	1	2	2	1
5550	1500	.2134	1	2	2	0
5550	1520	.2156	1	3	3	0
5550	1540	.2178	1	3	3	0
5550	1560	.2200	1	3	3	0
5550	1580	.2222	1	3	3	0
5550	1600	.2243	1	3	3	0
5600	1300	.1893	1	1	1	0
5600	1320	.1917	1	1	1	0
5600	1340	.1940	1	1	1	0
5600	1360	.1963	1	1	1	0
5600	1380	.1985	1	1	1	0
5600	1400	.2008	1	2	2	1
5600	1420	.2031	1	2	2	1
5600	1440	.2053	1	2	2	1
5600	1460	.2075	1	2	2	1
5600	1480	.2097	1	2	2	1
5600	1500	.2119	1	2	2	1
5600	1520	.2141	1	3	3	0
5600	1540	.2163	1	3	3	0
5600	1560	.2185	1	3	3	0
5600	1580	.2206	1	3	3	0
5600	1600	.2228	1	3	3	0
5650	1300	.1880	1	1	1	0
5650	1320	.1903	1	1	1	0
5650	1340	.1926	1	1	1	0
5650	1360	.1949	1	1	1	0
5650	1380	.1971	1	1	1	0
5650	1400	.1994	1	2	2	0
5650	1420	.2016	1	2	2	1
5650	1440	.2039	1	2	2	1
5650	1460	.2061	1	2	2	1
5650	1480	.2083	1	2	2	1
5650	1500	.2105	1	2	2	1

5650	1520	.2127	1	3	3	1
5650	1540	.2148	1	3	3	0
5650	1560	.2170	1	3	3	0
5650	1580	.2191	1	3	3	0
5650	1600	.2213	1	3	3	0
5700	1300	.1867	1	1	1	0
5700	1320	.1889	1	1	1	0
5700	1340	.1912	1	1	1	0
5700	1360	.1935	1	1	1	0
5700	1380	.1958	1	1	1	0
5700	1400	.1980	1	2	2	0
5700	1420	.2002	1	2	2	1
5700	1440	.2024	1	2	2	1
5700	1460	.2047	1	2	2	1
5700	1480	.2068	1	2	2	1
5700	1500	.2090	1	2	2	1
5700	1520	.2112	1	3	3	1
5700	1540	.2134	1	3	3	0
5700	1560	.2155	1	3	3	0
5700	1580	.2176	1	3	3	0
5700	1600	.2198	1	3	3	0
5750	1300	.1853	1	1	1	0
5750	1320	.1876	1	1	1	0
5750	1340	.1899	1	1	1	0
5750	1360	.1922	1	1	1	0
5750	1380	.1944	1	1	1	0
5750	1400	.1966	1	2	2	0
5750	1420	.1988	1	2	2	0
5750	1440	.2011	1	2	2	1
5750	1460	.2032	1	2	2	1
5750	1480	.2054	1	2	2	1
5750	1500	.2076	1	2	2	1
5750	1520	.2098	1	3	3	1
5750	1540	.2119	1	3	3	1
5750	1560	.2140	1	3	3	0
5750	1580	.2162	1	3	3	0
5750	1600	.2183	1	3	3	0

DISCUSSION

It has been demonstrated how a Markovian model can be applied to the determination of circuit reliability from data on the component characteristics. Component part reliability testing programs are in progress in many areas of the electronic industry in U.S.A. Today these programs vary in magnitude, levels of environmental and electrical stress and types of component part tested.

The Markovian model developed here can be of significant value to the semi-conductor electronics, especially in the wake of the aero-space age.

The model, however, has many shortcomings. We found that our computations were cumbersome and tedious even for a two parameter circuit. This leaves the model open to criticism so far as the practicability of the approach is concerned. A search for a simpler approach is needed, by the alteration of the model to suit our requirements. This can be brought about by an automated technique in the computer for,

(1) Partitioning the component data into states.

(2) Estimation and testing of Markov chains.

(3) Determination of the failure boundaries of the circuit.

(4) Computation of the reliability of the circuit at the desired time points.

Another shortcoming is that the model can only utilize data that comes from a first order, stationary, continuous time finite state process. Thus, it is necessary to verify at each time point the validity of the data with respect to the model.

Despite the many weaknesses displayed by the model, the approach is very useful in the reliability study of electronic components under environmental and electrical stresses.

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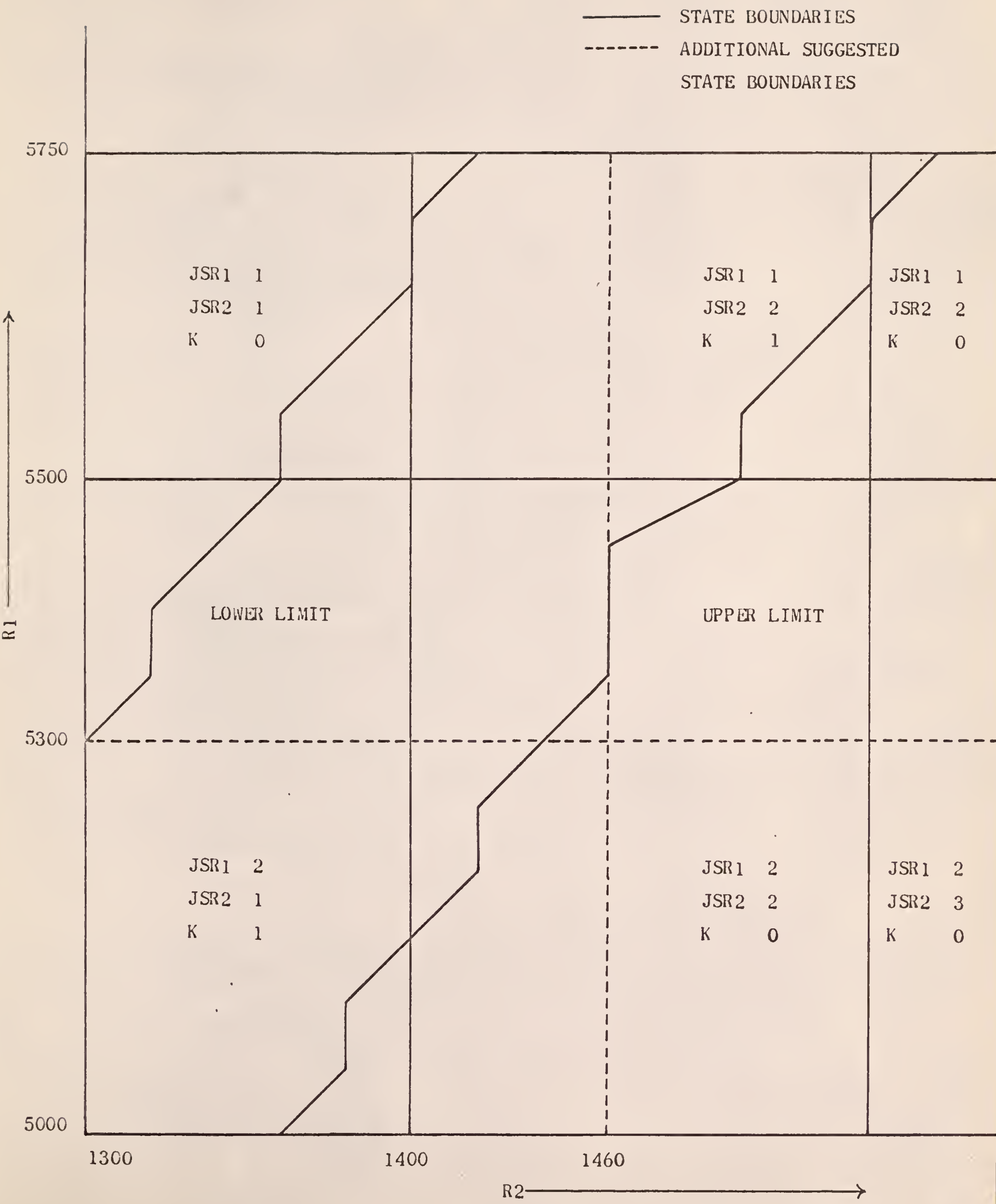
APPENDIX


STATE BOUNDARIES
ADDITIONAL SUGGESTED STATE BOUNDARIES
5750
5500
5300
5000
R1
JSR1 1
JSR2 1
K 0
JSR1 1
JSR2 2
K 1
JSR1 1
JSR2 2
K 0
LOWER LIMIT
UPPER LIMIT
JSR1 2
JSR2 1
K 1
JSR1 2
JSR2 2
K 0
JSR1 2
JSR2 3
K 0
1300
1400
1460
R2

This appendix has been devoted to a graphical approach of the failure boundary determination.

The following data were summarized from the computer output.

R2	The upper limit of value of R1 at which the circuit failed.	The lower limit of value of R1 at which the circuit failed.
5000	1360	--
5050	1380	--
5100	1380	--
5150	1400	--
5200	1420	--
5250	1420	1300
5300	1440	1300
5350	1460	1320
5400	1460	1320
5450	1460	1340
5500	1500	1360
5550	1500	1360
5600	1520	1380
5650	1540	1400
5700	1540	1400
5750	1560	1420

(1) These data are graphically plotted with ordinate 'R2' and abscissa 'R1.'

(2) Two plots are obtained namely for the upper and lower limits or R2 above or below which failure took place.

The region of success is obviously the area between the two plots.

(3) The boundaries of the states of R1 and R2 are clearly shown.

The objective will be to define the states, as to obtain the states of failure and success with definiteness. It is clear from the graph, that the present definition of states require revision. In fact, an additional state for each of the parameters R1 and R2 can be defined.

Another fact that is evident from the graph is that a large number of states for each of R1 and R2 is further helpful in a clear and accurate determination of the failure boundaries.

A MARKOVIAN RELIABILITY ANALYSIS
OF ELECTRONIC CIRCUITS

by

M. SUNDARARAMAN

B.S., Ranchi University, India, 1962
(Electrical Engineering)

AN ABSTRACT OF A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Industrial Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1965

The report deals with the evaluation of reliability of electronic circuits from the knowledge of data on component parts. This study has been motivated by the fact that a reliability model must include two forms of failures. Namely, failure of the component by the catastrophic failure of the parameters, and failure of the component by the drift of the component parameters. A Markovian reliability model is found to provide good results for the reliability of an electronic circuit that incorporates both these forms of failures. A generalized expression for the reliability of a circuit containing 'r' parameters is obtained. Based on this approach, the reliability of a specific two resistor circuit is evaluated.

A continuous Markovian representation of the component parameter drift is based on the assumption that the state of the component parameter at some future time, given the state of the component parameter time is independent of the present age of the component, the previous states occupied, the time spent in each of the previous states, and the time spent in the present state. An analysis of this nature is believed to partially fill the need for an exhaustive reliability study of electronic circuits.